CHAPTER

1 The Smith predictor, the modified Smith predictor and the finite spectrum assignment: a comparative study

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ABSTRACT
This chapter deals with predictor feedback controllers to compensate time delays in feedback loops. The concept and the governing equations of the Smith predictor, the modified Smith predictor and the finite spectrum assignment are discussed in detail. The relationship between the three control strategies is established both in frequency and time domain, and a detailed comparison is performed with respect to the properties of the closed control loop. Both the Smith predictor and the finite spectrum assignment is based on the prediction of the state at time instant $t+\tau$ with τ being the feedback delay. In this chapter, it is argued that the main difference is that the Smith predictor involves a model to be solved over the entire time interval $[0, t+\tau]$, while the finite spectrum assignment employs an internal model only over the delay period $[t, t+\tau]$. It is also shown that the governing equations behind the modified Smith predictor and the finite spectrum assignment are equivalent and the difference between the two concepts lies in their implementation. Issues related to practical realization are also discussed including the effect of uncertainties in the parameters and in the initial conditions, the implementation of the control law, and the utilization of observers.

Keywords: Time-delay systems, control theory, predictor feedback, Smith predictor, modified Smith predictor, finite spectrum assignment

1.1 INTRODUCTION

Feedback loops in control systems are always associated with time delays due to the finite speed of sensing, signal processing, computation of the control input and actuation. Feedback delay is usually considered to be a source of unstable behavior, which should be eliminated from the control loop. An effective way to compensate the destabilizing effect of feedback delays is the application of predictor feedback controllers such as the Smith predictor [1] and its modifications [2, 3, 4], the prediction based on optimal control [5], the finite spectrum assignment [6, 7, 8, 9], the reduction approach [10] or the predictive pole placement control [11]. An in-depth discussion on time delay compensation is given in [12].

This chapter gives a tutorial overview of the well-known Smith predictor (SP), the modified Smith predictor (MSP) and the finite spectrum assignment (FSA) technique. Throughout the chapter, the works of Zhong [13] and Michiels and Niculescu [8] are followed and extended by time-domain equations. The core idea of these predictor feedback controllers is to estimate (predict) the future state (or output) of the plant. Using the predicted state (or output) in the feedback loop instead of the actual one, the effect of time delay can be compensated: an accurate prediction is able to completely eliminate the delay from the feedback loop. However, prediction requires an internal model of the system that allows the calculation of the predicted state (or output). Inaccuracies of this model and imperfections in the implementation of the predictor affect the control performance significantly. Hereinafter, the basic approach and the implementation of the SP, the MSP and the FSA are overviewed.

In the literature, the difference between the SP and the FSA is often attributed to the observer-predictor or predictor-observer representations [13, 14, 15]. Here, we argue that the difference rather lies in the method of prediction. The Smith predictor employs a prediction over the time interval $[0, t + \tau]$ with τ being the feedback delay, while the finite spectrum assignment predicts only over the delay period $[t, t + \tau]$.

In Section 1.2, the control laws of the SP, the MSP and the FSA techniques are summarized. The governing equations of the closed control loop are shown both in frequency and time domain, and block diagrams are given. Based on the equations, the relationship between these control strategies is established and the most important differences are pointed out in Section 1.3. It is highlighted that initial conditions (and disturbances) affect the performance of the SP and the MSP, but do not affect the closed-loop behavior for the FSA technique. Then, it is shown that the governing equations of the MSP and the FSA are practically equivalent, and the difference between them lies in the formulation and the realization of their control laws. Issues related to practical realization are considered in terms of sensitivity to parameter mismatches and initial conditions and implementation of the control laws. In Section 1.4, the application of observers is discussed, which is a necessary step when the state of the system is not fully available for the predictor. The governing equations of observer-predictor and predictor-observer representations are given and their connection is established. Finally, a summary is given in Section 1.5.

1.2 DESCRIPTION OF PREDICTOR FEEDBACK CONTROLLERS

This chapter is devoted to the analysis of single input-single output systems with discrete input delay. For extensions to multiple inputs, multiple, distributed and varying delays, see [12, 16].

1.2.1 CONTROL PROBLEM WITHOUT PREDICTOR

Consider the control problem illustrated in the block diagram of Figure 1.1(a). The control input u is used in order to adjust the output y of the plant to the reference signal r in the presence of a disturbance d. Unless stated otherwise, we neglect the effect of the disturbance d, although comments will be made on disturbance response later in this section. We assume that the input u is subjected to a single point delay τ as indicated by the term $\mathrm{e}^{-s\tau}$ in the block diagram. The transfer function of the corresponding delay-free plant is indicated by $P(s)$, while the transfer function of the controller is denoted by $C(s)$. Taking the Laplace transforms with zero initial conditions and $d = 0$, we obtain

$$Y(s) = P(s)\mathrm{e}^{-s\tau}U(s)\,, \tag{1.1}$$

$$U(s) = C(s)(R(s) - Y(s))\,, \tag{1.2}$$

where $Y(s)$, $U(s)$ and $R(s)$ denote the Laplace transform of $y(t)$, $u(t)$ and $r(t)$, respectively. For notational convenience, capital letters are used to indicate frequency-domain quantities.

The transfer function of the closed control loop from the reference signal r to the output y becomes

$$T(s) = \frac{C(s)P(s)\mathrm{e}^{-s\tau}}{1 + C(s)P(s)\mathrm{e}^{-s\tau}}\,. \tag{1.3}$$

It can be seen that the denominator of the transfer function involves the delay, which affects the stability of the closed control loop. Since time delays are typically a source of unstable behavior, they should be eliminated from the feedback loop. In this chapter, this problem is addressed by predictor feedback controllers such as the Smith predictor, the modified Smith predictor and the finite spectrum assignment.

The delay-free plant $P(s)$ can also be described by the state-space representation

$$P(s) = \left[\begin{array}{c|c} \mathbf{A} & \mathbf{B} \\ \hline \mathbf{C} & 0 \end{array}\right] = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\,, \tag{1.4}$$

where $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\mathbf{B} \in \mathbb{R}^{n\times 1}$ and $\mathbf{C} \in \mathbb{R}^{1\times n}$ are the system, input and output matrices, respectively, n is the number of state variables, and $\mathbf{I}$ is the identity matrix. Note that this model could easily be extended to a multi input-multi output case, but for simplicity we restrict ourselves to the model presented above. Throughout this chapter,

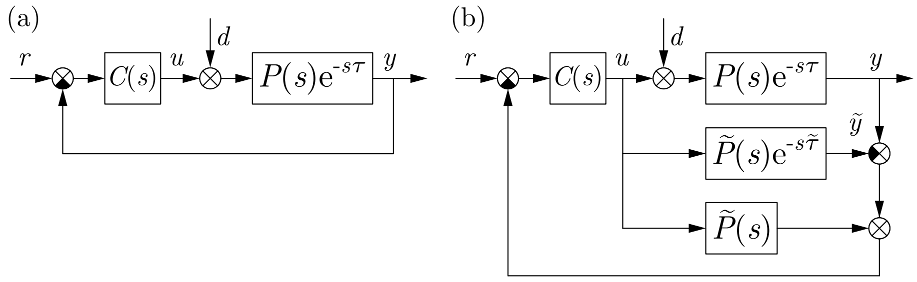

FIGURE 1.1

Block diagram of control loops without predictors (a); block diagram of the Smith predictor (b).

we assume that the pair $(\mathbf{A}, \mathbf{B})$ is controllable, while the pair $(\mathbf{C}, \mathbf{A})$ is observable. The state-space representation of the plant $P(s)\mathrm{e}^{-s\tau}$ in time domain is given by

$$\begin{aligned} \dot{\mathbf{x}}(t) =& \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t-\tau)\,, \\ y(t) =& \mathbf{C}\mathbf{x}(t)\,, \end{aligned} \tag{1.5}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the vector of state variables.

In order to establish the relationship between the different predictor feedback controllers in time domain, we consider full state feedback and proportional (static) output feedback. Note that full state feedback requires the state of the system to be available for control. If this is not the case, either output feedback can be used or observers can be applied (which is addressed later in Section 1.4). The control law of delayed state feedback controllers reads

$$u(t) = \mathbf{K}\mathbf{x}(t)\,, \tag{1.6}$$

where $\mathbf{K} \in \mathbb{R}^{1\times n}$ is a feedback matrix. For the special case $\mathbf{K} = k\mathbf{C}$, delayed state feedback reduces to proportional delayed output feedback

$$u(t) = ky(t)\,. \tag{1.7}$$

In these time-domain equations, the disturbance d and the reference signal r are assumed to be zero in order to simplify the analysis. Of course, nonzero reference signal and nonzero disturbance could also be taken into account. Furthermore, the control gain k could also be replaced by any controller given by the transfer function $C(s)$.

Note that the Smith predictor and the modified Smith predictor are typically introduced as output predictors with output feedback, while the finite spectrum assignment is usually formulated for state predictors and state feedback. Extensions to state and output feedback, respectively, can easily be done as shown below.

1.2.2 THE SMITH PREDICTOR

The Smith predictor (SP) is intended to compensate the destabilizing effect of the delay τ in order to achieve the delayed response of the delay-free system [13]. The approach utilizes an internal model $\tilde{P}(s)\mathrm{e}^{-s\tilde{\tau}}$ of the plant $P(s)\mathrm{e}^{-s\tau}$ to predict the future behavior of the system. Note that the dynamics of the plant is never known accurately, there are always mismatches between the actual plant $P(s)$ and its model $\tilde{P}(s)$, as well as between the actual delay τ and its estimation $\tilde{\tau}$. Estimation quantities are indicated by tilde throughout the chapter. Therefore, only an estimation $\tilde{y}$ of the output y can be obtained via the internal model:

$$\tilde{Y}(s) = \tilde{P}(s)\mathrm{e}^{-s\tilde{\tau}}U(s)\,. \tag{1.8}$$

The future output can also be estimated (predicted) by $\tilde{Y}(s)\mathrm{e}^{s\tilde{\tau}}$, which is no longer subjected to the input delay. This can be used to correct the output to its predicted value in the control law:

$$U(s) = C(s)\Big(R(s) - \Big(Y(s) - \tilde{Y}(s) + \tilde{Y}(s)\mathrm{e}^{s\tilde{\tau}}\Big)\Big), \tag{1.9}$$

cf. Equation (1.2). This control law can be realized by the term $Z(s)$ as

$$\begin{aligned} Z(s) &= \tilde{P}(s) - \tilde{P}(s)\mathrm{e}^{-s\tilde{\tau}}\,, \\ U(s) &= C(s)\left(R(s) - \left(Y(s) + Z(s)U(s)\right)\right), \end{aligned} \tag{1.10}$$

that is illustrated by the block diagram in Figure 1.1(b).

Finally, the controller with the SP can be described by

$$C_{\mathrm{SP}}(s) = \frac{C(s)}{1 + C(s)Z(s)} = \frac{C(s)}{1 + C(s)\tilde{P}(s) - C(s)\tilde{P}(s)\mathrm{e}^{-s\tilde{\tau}}}\,, \tag{1.11}$$

by which the closed-loop transfer function becomes

$$T_{\mathrm{SP}}(s) = \frac{C_{\mathrm{SP}}(s)P(s)\mathrm{e}^{-s\tau}}{1 + C_{\mathrm{SP}}(s)P(s)\mathrm{e}^{-s\tau}} = \frac{C(s)P(s)\mathrm{e}^{-s\tau}}{1 + C(s)\tilde{P}(s) - C(s)\tilde{P}(s)\mathrm{e}^{-s\tilde{\tau}} + C(s)P(s)\mathrm{e}^{-s\tau}}\,. \tag{1.12}$$

For the ideal case with perfectly accurate estimation of the plant dynamics and the delay ($\tilde{P}(s) = P(s)$, $\tilde{\tau} = \tau$), the transfer function simplifies to

$$T_{\mathrm{SP}}^{\mathrm{id}}(s) = \frac{C(s)P(s)\mathrm{e}^{-s\tau}}{1 + C(s)P(s)}\,, \tag{1.13}$$

cf. Equation (1.3). This means that the SP is able to remove the effect of the time delay on the poles of the closed control loop, and achieves the delayed response of the delay-free plant $P(s)$ subjected to controller $C(s)$. In reality, however, a perfect internal model ($\tilde{P}(s) = P(s)$, $\tilde{\tau} = \tau$) is never achievable, thus the delay cannot be completely eliminated from the control loop. Control performance depends on the mismatches between the internal model and the actual system [17].

Considering disturbance response, the transfer function from the disturbance d to the output y reads

$$W_{\mathrm{SP}}(s) = \frac{P(s)\mathrm{e}^{-s\tau}}{1 + C_{\mathrm{SP}}(s)P(s)\mathrm{e}^{-s\tau}} = \frac{\left(1 + C(s)\tilde{P}(s) - C(s)\tilde{P}(s)\mathrm{e}^{-s\tilde{\tau}}\right)C(s)P(s)\mathrm{e}^{-s\tau}}{1 + C(s)\tilde{P}(s) - C(s)\tilde{P}(s)\mathrm{e}^{-s\tilde{\tau}} + C(s)P(s)\mathrm{e}^{-s\tau}}\,, \tag{1.14}$$

which, in the ideal case $\tilde{P}(s) = P(s)$, $\tilde{\tau} = \tau$, becomes

$$W_{\mathrm{SP}}^{\mathrm{id}}(s) = \frac{(1 + C(s)P(s) - C(s)P(s)\mathrm{e}^{-s\tau})\,C(s)P(s)\mathrm{e}^{-s\tau}}{1 + C(s)P(s)}\,. \tag{1.15}$$

This implies that, in the ideal case, the poles of the disturbance response involve those of the plant $P(s)$. Usually this argument is used to explain the incapability of the SP to stabilize unstable plants. Note, however, when $\tilde{P}(s) \neq P(s)$ and $\tilde{\tau} \neq \tau$, both the actual plant $P(s)$ and the model $\tilde{P}(s)$ affect the poles (and thus the stability) of the closed control loop, which opens the possibility of stabilization for some extreme model parameter mismatches (for further details, see [17]).

The delay-free internal model $\tilde{P}(s)$ can be represented in state-space form by

$$\tilde{P}(s) = \left[\begin{array}{c|c} \tilde{\mathbf{A}} & \tilde{\mathbf{B}} \\ \hline \tilde{\mathbf{C}} & 0 \end{array}\right] = \tilde{\mathbf{C}}(s\mathbf{I} - \tilde{\mathbf{A}})^{-1}\tilde{\mathbf{B}}\,, \tag{1.16}$$

where $\tilde{\mathbf{A}}$, $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{C}}$ are the estimations (nominal values) of matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$. Accordingly, the internal model $\tilde{P}(s)\mathrm{e}^{-s\tilde{\tau}}$ is represented in time domain as

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}(t) &= \tilde{\mathbf{A}}\tilde{\mathbf{x}}(t) + \tilde{\mathbf{B}}u(t - \tilde{\tau})\,, \\ \tilde{y}(t) &= \tilde{\mathbf{C}}\tilde{\mathbf{x}}(t)\,, \end{aligned} \tag{1.17}$$

where $\tilde{\mathbf{x}}$ is the estimation of the state $\mathbf{x}$ given by Equation (1.5). In the case of proportional output feedback, the control law of the SP reads

$$u(t) = k\big(y(t) - \tilde{y}(t) + \tilde{y}(t + \tilde{\tau})\big)\,, \tag{1.18}$$

while for a state feedback controller it becomes

$$u(t) = \mathbf{K}\big(\mathbf{x}(t) - \tilde{\mathbf{x}}(t) + \tilde{\mathbf{x}}(t + \tilde{\tau})\big)\,. \tag{1.19}$$

This time-domain representation of the SP was given in [12] (see Equation (2.45) in [12]). Note that if $\tilde{\mathbf{C}} = \mathbf{C}$, proportional output feedback is achieved by the choice $\mathbf{K} = k\mathbf{C}$. The block diagram of the SP with state-space representation and state feedback is illustrated in Figure 1.2(a).

1.2.3 THE MODIFIED SMITH PREDICTOR

The fact that the internal model $\tilde{P}(s)$ used by the predictor affects the stability of the closed control loop gives motivation for an improvement of this model. Here, a specific modified model $\hat{P}(s)$ is used, which is no longer simply an estimation of the plant $P(s)$. The modified Smith predictor (MSP) [13] employs a modified model

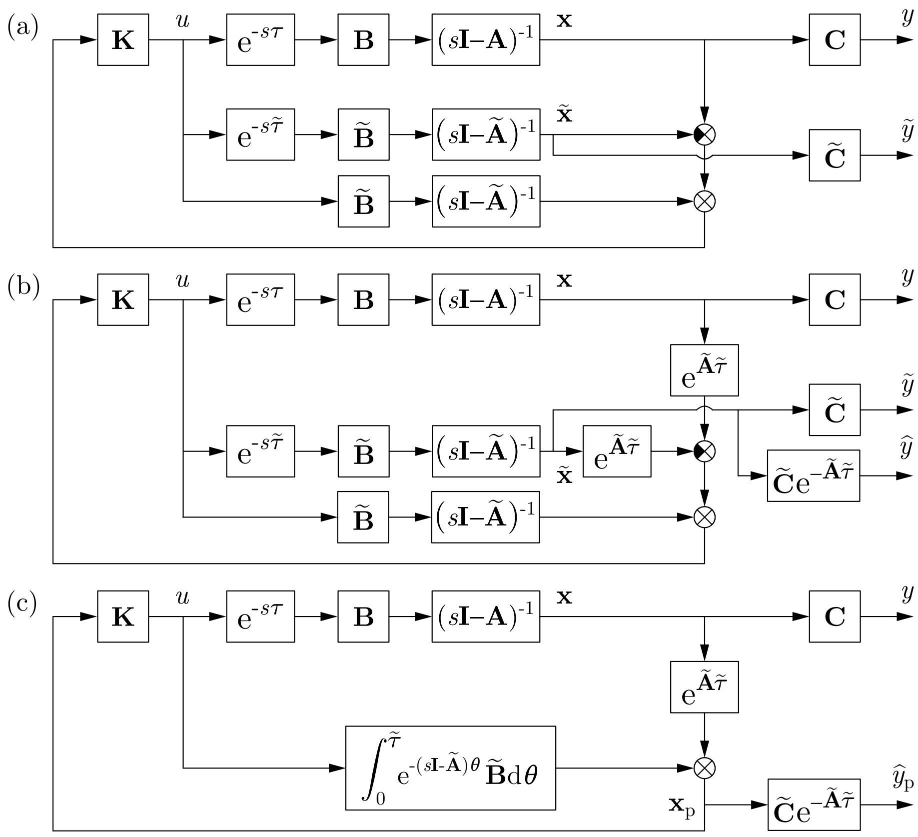

FIGURE 1.2

Block diagram of the Smith predictor (a); the modified Smith predictor (b); and the finite spectrum assignment (c) in the case of state feedback.

$\hat{P}(s)$, which is based on state-space representation:

$$\hat{P}(s) = \left[\begin{array}{c|c} \tilde{\mathbf{A}} & \tilde{\mathbf{B}} \\ \hline \tilde{\mathbf{C}}\mathrm{e}^{-\tilde{\mathbf{A}}\tilde{\tau}} & 0 \end{array}\right] = \tilde{\mathbf{C}}\mathrm{e}^{-\tilde{\mathbf{A}}\tilde{\tau}}(s\mathbf{I} - \tilde{\mathbf{A}})^{-1}\tilde{\mathbf{B}}\,. \tag{1.20}$$

The corresponding modified quantities are indicated by hat throughout the chapter. This way, the predictor and the control law become

$$\begin{aligned} \hat{Z}(s) &= \hat{P}(s) - \tilde{P}(s)\mathrm{e}^{-s\tilde{\tau}}\,, \\ U(s) &= C(s)\Big(R(s) - \big(Y(s) + \hat{Z}(s)U(s)\big)\Big)\,. \end{aligned} \tag{1.21}$$

The controller involving the MSP can be described by

$$C_{\text{MSP}}(s) = \frac{C(s)}{1 + C(s)\hat{Z}(s)}\,, \tag{1.22}$$

while the transfer function from the reference signal to the output becomes

$$T_{\text{MSP}}(s) = \frac{C_{\text{MSP}}(s)P(s)\mathrm{e}^{-s\tau}}{1 + C_{\text{MSP}}(s)P(s)\mathrm{e}^{-s\tau}}\,, \tag{1.23}$$

cf. Equations (1.11) and (1.12). Note that since typically $\hat{P}(0) \neq \tilde{P}(0)$, one may use $\hat{Z}(s) - \hat{Z}(0)$ instead of $\hat{Z}(s)$ in the second row of Equation (1.21) in order to guarantee zero static error for the output [13].

In time domain, the modified model $\hat{P}(s)\mathrm{e}^{-s\tilde{\tau}}$ with delay is obtained by

$$\hat{y}(t) = \tilde{\mathbf{C}}\mathrm{e}^{-\tilde{\mathbf{A}}\tilde{\tau}}\tilde{\mathbf{x}}(t)\,, \tag{1.24}$$

where the estimated state $\tilde{\mathbf{x}}$ is governed by Equation (1.17). Note that since $\tilde{\mathbf{A}}\mathrm{e}^{-\tilde{\mathbf{A}}\tilde{\tau}} = \mathrm{e}^{-\tilde{\mathbf{A}}\tilde{\tau}}\tilde{\mathbf{A}}$, one may introduce the modified state

$$\hat{\mathbf{x}}(t) = \mathrm{e}^{-\tilde{\mathbf{A}}\tilde{\tau}}\tilde{\mathbf{x}}(t) \tag{1.25}$$

and rewrite Equations (1.17) and (1.24) in the form

$$\begin{aligned} \dot{\hat{\mathbf{x}}}(t) &= \tilde{\mathbf{A}}\hat{\mathbf{x}}(t) + \mathrm{e}^{-\tilde{\mathbf{A}}\tilde{\tau}}\tilde{\mathbf{B}}u(t - \tilde{\tau})\,, \\ \hat{y}(t) &= \tilde{\mathbf{C}}\hat{\mathbf{x}}(t)\,. \end{aligned} \tag{1.26}$$

Accordingly, the control law for proportional output feedback reads

$$u(t) = \hat{k}\big(y(t) - \tilde{y}(t) + \hat{y}(t + \tilde{\tau})\big)\,, \tag{1.27}$$

where the gain $\hat{k}$ could be replaced by other controllers given by the transfer function $C(s)$. The corresponding state feedback controller with feedback matrix $\hat{\mathbf{K}}$ is

$$u(t) = \hat{\mathbf{K}}\big(\mathbf{x}(t) - \tilde{\mathbf{x}}(t) + \hat{\mathbf{x}}(t + \tilde{\tau})\big)\,, \tag{1.28}$$

which reduces to the case of proportional output feedback with $\hat{\mathbf{K}} = \hat{k}\mathbf{C}$ if $\tilde{\mathbf{C}} = \mathbf{C}$. Note that this control law can also be expressed by the estimated state $\tilde{\mathbf{x}}$ instead of the modified state $\hat{\mathbf{x}}$ as

$$u(t) = \mathbf{K}\left(\mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}(\mathbf{x}(t) - \tilde{\mathbf{x}}(t)) + \tilde{\mathbf{x}}(t + \tilde{\tau})\right), \tag{1.29}$$

where $\hat{\mathbf{K}} = \mathbf{K}\mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}$. This shows that the difference between the SP and the MSP techniques is the term $\mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}$, which plays an important role considering the effect of initial conditions in Section 1.3.1. The block diagram corresponding to the MSP is the one in Figure 1.1(b) with $\hat{P}(s)$ instead of $\tilde{P}(s)$. A more detailed block diagram with the state-space representation is illustrated in Figure 1.2(b).

1.2.4 FINITE SPECTRUM ASSIGNMENT

The finite spectrum assignment (FSA) concept is originated from time-domain representation. The classical form of FSA is developed to compensate the input delay for the state feedback given by Equation (1.6), see [6, 7, 9]. Similarly to the SP and the MSP techniques, FSA intends to use a predicted value $\mathbf{x}_\mathrm{p}(t+\tilde{\tau})$ of the state instead of using the actual one $\mathbf{x}(t)$ for feedback. Again, an internal model is used to perform the prediction

$$\dot{\mathbf{x}}_\mathrm{p}(t) = \tilde{\mathbf{A}}\mathbf{x}_\mathrm{p}(t) + \tilde{\mathbf{B}}u(t-\tilde{\tau})\,, \tag{1.30}$$

cf. Equation (1.17). The state $\mathbf{x}(t)$ is used as initial condition, and the internal model is solved by the predictor over the estimated delay interval $[t, t+\tilde{\tau}]$. This leads to the predicted state

$$\mathbf{x}_\mathrm{p}(t+\tilde{\tau}) = \mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}\mathbf{x}(t) + \int_0^{\tilde{\tau}} \mathrm{e}^{\tilde{\mathbf{A}}\theta}\tilde{\mathbf{B}}u(t-\theta)\mathrm{d}\theta\,, \tag{1.31}$$

which is used for state feedback:

$$u(t) = \mathbf{K}\mathbf{x}_\mathrm{p}(t+\tilde{\tau})\,. \tag{1.32}$$

Consider the case of an ideal internal model (1.30) and an accurate estimation of the delay: $\tilde{\mathbf{A}} = \mathbf{A}$, $\tilde{\mathbf{B}} = \mathbf{B}$ and $\tilde{\tau} = \tau$. Then, Equations (1.31) and (1.32) can be simplified as follows. Substitute $\mathbf{B}u(t-\theta)$ from Equation (1.5) into Equation (1.31) and use the equality $\dot{\mathbf{x}}(t+\tau-\theta) = -\mathbf{x}'(t+\tau-\theta)$ (where prime denotes differentiation with respect to θ). Then, integration by parts leads to

$$u(t) = \mathbf{K}\mathbf{x}(t+\tau)\,. \tag{1.33}$$

Thus, Equations (1.5) and (1.33) describe a delay-free state feedback, and the closed control loop is associated with an ordinary differential equation. This implies that FSA eliminates the delay from the control loop in case of a perfectly accurate internal model. The spectrum (the set of poles) of the closed-loop system becomes finite, which can be assigned via the control parameters in $\mathbf{K}$. Thus, stability can be achieved for arbitrary (but controllable) pairs of $(\mathbf{A}, \mathbf{B})$ and arbitrary delay τ. That is, even an unstable plant can be stabilized by FSA. Elimination of the delay requires, however, that the parameters of the internal model (1.30) match those of the actual system (1.5) and that the control law (1.32) is implemented accurately. The effects of implementation inaccuracies are addressed in Section 1.3.3, while sensitivity to parameter mismatches is analyzed in [18].

FSA can also be formulated for output feedback following [8]. Note that the output $y_\mathrm{p}(t+\tilde{\tau}) = \tilde{\mathbf{C}}\mathbf{x}_\mathrm{p}(t+\tilde{\tau})$ requires the knowledge of the state $\mathbf{x}(t)$ according to Equation (1.31). If the state is not fully known, the following output can be introduced:

$$\hat{y}_\mathrm{p}(t+\tilde{\tau}) = \tilde{\mathbf{C}}\mathrm{e}^{-\tilde{\mathbf{A}}\tilde{\tau}}\mathbf{x}_\mathrm{p}(t+\tilde{\tau})\,, \tag{1.34}$$

cf. Equation (1.24) for the MSP. The coefficient $\mathrm{e}^{-\tilde{\mathbf{A}}\tilde{\tau}}$ allows us to write Equations (1.31) and (1.34) in the form

$$\hat{y}_{\text{p}}(t + \tilde{\tau}) = y(t) + \tilde{\mathbf{C}}\text{e}^{-\tilde{\mathbf{A}}\tilde{\tau}} \int_0^{\tilde{\tau}} \text{e}^{\tilde{\mathbf{A}}\theta}\tilde{\mathbf{B}}u(t - \theta)\text{d}\theta\,, \tag{1.35}$$

where the available output $y(t)$ is used instead of the unavailable term $\tilde{\mathbf{C}}\mathbf{x}(t)$. Via Equation (1.35), the predicted output $\hat{y}_{\text{p}}(t + \tilde{\tau})$ can be calculated based on the output $y(t)$ without knowing the full state $\mathbf{x}(t)$. The control law for proportional output feedback becomes

$$u(t) = \hat{k}\hat{y}_{\text{p}}(t + \tilde{\tau})\,. \tag{1.36}$$

For details on other (dynamic) output feedback controllers, see [8].

Via Laplace transformation, the description of FSA in frequency domain is the following. The integral term in the predictors (1.31) and (1.35) is represented by

$$\mathbf{Z}_x(s) = \int_0^{\tilde{\tau}} \text{e}^{-(s\mathbf{I}-\tilde{\mathbf{A}})\theta}\tilde{\mathbf{B}}\text{d}\theta = \left(\mathbf{I} - \text{e}^{-(s\mathbf{I}-\tilde{\mathbf{A}})\tilde{\tau}}\right)(s\mathbf{I} - \tilde{\mathbf{A}})^{-1}\tilde{\mathbf{B}}\,, \tag{1.37}$$

by which the predicted state and the predicted output become

$$\mathbf{X}_{\text{p}}(s)\text{e}^{s\tilde{\tau}} = \text{e}^{\tilde{\mathbf{A}}\tilde{\tau}}\mathbf{X}(s) + \mathbf{Z}_x(s)U(s)\,, \tag{1.38}$$

$$\hat{Y}_{\text{p}}(s)\text{e}^{s\tilde{\tau}} = Y(s) + \tilde{\mathbf{C}}\text{e}^{-\tilde{\mathbf{A}}\tilde{\tau}}\mathbf{Z}_x(s)U(s)\,. \tag{1.39}$$

The controller corresponding to Equations (1.10) and (1.21) is therefore

$$U(s) = C(s)\left(R(s) - \hat{Y}_{\text{p}}(s)\text{e}^{s\tilde{\tau}}\right). \tag{1.40}$$

The block diagram associated with a state feedback controller is shown in Figure 1.2(c). Note that in the case of an unstable plant, Equation (1.37) cannot be used directly to realize the control law, see the discussion in Section 1.3.3.

1.3 COMPARISON OF THE PREDICTORS

For comparison, a summary of the time-domain governing equations for the SP, the MSP and the FSA are listed in Table 1.1 at the end of the chapter. Respectively, the corresponding frequency-domain equations are listed in Table 1.2. Some key differences between the different predictor concepts are highlighted below.

1.3.1 EFFECT OF INITIAL CONDITIONS

Note that for the SP and the MSP, the actual state $\mathbf{x}(t)$ is corrected by subtracting the model state $\tilde{\mathbf{x}}(t)$ and adding either the predicted state $\tilde{\mathbf{x}}(t + \tilde{\tau})$ or the modified predicted state $\hat{\mathbf{x}}(t + \tilde{\tau})$, respectively, as shown by Equations (1.19) and (1.28). In contrast, FSA directly uses the predicted state $\mathbf{x}_{\text{p}}(t + \tilde{\tau})$, which is obtained from the internal model with the initial condition $\mathbf{x}_{\text{p}}(t) = \mathbf{x}(t)$. This shows a significant difference between the Smith predictor concepts and the FSA approach: the initial conditions of the internal model are taken at different time instants.

In what follows, we investigate the effect of initial conditions for the state feedback controllers (1.19), (1.29) and (1.32). For the sake of simplicity, we study the following initial conditions for the actual system (1.5) and the model (1.17), respectively: $\mathbf{x}(0) = \mathbf{x}_0$, $\mathbf{x}(t) \equiv \mathbf{0}$ for $t < 0$ and $\tilde{\mathbf{x}}(0) = \tilde{\mathbf{x}}_0$, $\tilde{\mathbf{x}}(t) \equiv \mathbf{0}$ for $t < 0$. Note, however, that the conclusions drawn in this section hold for a general initial condition, too. Since the initial conditions of the actual system (1.5) are unknown to the controller, they can only be estimated by those of the model (1.17). There are always mismatches between the initial conditions ($\tilde{\mathbf{x}}_0 \neq \mathbf{x}_0$), whose effect is shown below.

By solving Equations (1.5) and (1.17), controller (1.19) for the SP can be given in the form

$$u(t) = \mathbf{K}\bigg(\mathrm{e}^{\mathbf{A}t}\mathbf{x}_0 + \int_0^t \mathrm{e}^{\mathbf{A}(t-\theta)}\mathbf{B}u(\theta-\tau)\mathrm{d}\theta - \mathrm{e}^{\tilde{\mathbf{A}}t}\tilde{\mathbf{x}}_0 - \int_0^t \mathrm{e}^{\tilde{\mathbf{A}}(t-\theta)}\tilde{\mathbf{B}}u(\theta-\tilde{\tau})\mathrm{d}\theta + \mathrm{e}^{\tilde{\mathbf{A}}(t+\tilde{\tau})}\tilde{\mathbf{x}}_0 + \int_0^{t+\tilde{\tau}} \mathrm{e}^{\tilde{\mathbf{A}}(t+\tilde{\tau}-\theta)}\tilde{\mathbf{B}}u(\theta-\tilde{\tau})\mathrm{d}\theta\bigg). \quad (1.41)$$

With perfectly matching internal model ($\tilde{\mathbf{A}} = \mathbf{A}$, $\tilde{\mathbf{B}} = \mathbf{B}$, $\tilde{\tau} = \tau$), this simplifies to

$$\begin{aligned} u(t) &= \mathbf{K}\left(\mathrm{e}^{\mathbf{A}t}\mathbf{x}_0 - \mathrm{e}^{\mathbf{A}t}\tilde{\mathbf{x}}_0 + \mathrm{e}^{\mathbf{A}(t+\tau)}\tilde{\mathbf{x}}_0 + \mathbf{x}(t+\tau) - \mathrm{e}^{\mathbf{A}(t+\tau)}\mathbf{x}_0\right) \\ &= \mathbf{K}\mathbf{x}(t+\tau) + \mathbf{K}\left(\mathrm{e}^{\mathbf{A}t} - \mathrm{e}^{\mathbf{A}(t+\tau)}\right)(\mathbf{x}_0 - \tilde{\mathbf{x}}_0). \end{aligned} \quad (1.42)$$

This shows that mismatches in the initial conditions directly affect the control input. For an unstable plant, the last term in Equation (1.42) tends to infinity as $t \to \infty$ if $\tilde{\mathbf{x}}_0 \neq \mathbf{x}_0$. (Note that the same could be shown for output feedback via the substitution $\mathbf{K} = k\mathbf{C}$.) This explains the incapability of the SP to stabilize unstable plants even in the case when the internal model is free of parameter mismatches. Recall that a similar conclusion was drawn considering the disturbance response in frequency domain in Section 1.2.2.

In a similar manner, initial conditions can be taken into account for the MSP. By solving Equations (1.5) and (1.17), the state feedback controller (1.29) becomes

$$u(t) = \mathbf{K}\bigg(\mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}\mathrm{e}^{\mathbf{A}t}\mathbf{x}_0 + \mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}\int_0^t \mathrm{e}^{\mathbf{A}(t-\theta)}\mathbf{B}u(\theta-\tau)\mathrm{d}\theta - \mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}\mathrm{e}^{\tilde{\mathbf{A}}t}\tilde{\mathbf{x}}_0 - \mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}\int_0^t \mathrm{e}^{\tilde{\mathbf{A}}(t-\theta)}\tilde{\mathbf{B}}u(\theta-\tilde{\tau})\mathrm{d}\theta + \mathrm{e}^{\tilde{\mathbf{A}}(t+\tilde{\tau})}\tilde{\mathbf{x}}_0 + \int_0^{t+\tilde{\tau}} \mathrm{e}^{\tilde{\mathbf{A}}(t+\tilde{\tau}-\theta)}\tilde{\mathbf{B}}u(\theta-\tilde{\tau})\mathrm{d}\theta\bigg). \quad (1.43)$$

A perfect internal model ($\tilde{\mathbf{A}} = \mathbf{A}$, $\tilde{\mathbf{B}} = \mathbf{B}$, $\tilde{\tau} = \tau$) leads to

$$u(t) = \mathbf{K}\left(\mathrm{e}^{\mathbf{A}(t+\tau)}\mathbf{x}_0 - \mathrm{e}^{\mathbf{A}(t+\tau)}\tilde{\mathbf{x}}_0 + \mathrm{e}^{\mathbf{A}(t+\tau)}\tilde{\mathbf{x}}_0 + \mathbf{x}(t+\tau) - \mathrm{e}^{\mathbf{A}(t+\tau)}\mathbf{x}_0\right) = \mathbf{K}\mathbf{x}(t+\tau). \quad (1.44)$$

For the MSP with perfect internal model, the effect of mismatches between the initial conditions drops owing to the term $\mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}$ in the controller. This enables the MSP to stabilize unstable plants despite the presence of mismatches between the initial conditions. Note, however, that for an imperfect internal model ($\tilde{\mathbf{A}} \neq \mathbf{A}$, $\tilde{\mathbf{B}} \neq \mathbf{B}$, $\tilde{\tau} \neq \tau$), the effect of initial conditions does not vanish. Therefore, the realization (1.29) of the MSP may lead to instabilities that are associated with unstable pole-zero cancellations. Thus, special care must be taken when implementing the control law – for details, see Section 1.3.3.

For FSA, the state $\mathbf{x}(t)$ of the plant is used in Equation (1.31) to obtain the predicted state $\mathbf{x}_\mathrm{p}(t+\tilde{\tau})$. Thus, prediction is done over the delay interval of length $\tilde{\tau}$ only and not over the whole time interval $[0, t+\tilde{\tau}]$. This way, the problem of mismatches between the initial conditions does not show up.

1.3.2 EQUIVALENCE OF THE MSP AND THE FSA

Note that the same delay-free control law is obtained for the FSA and for the MSP in the ideal case without parameter mismatches, cf. Equations (1.33) and (1.44). In fact, a more general equivalence holds for the control laws of these two techniques. Similarly to Equation (1.31), the internal model (1.17) can be solved over the delay interval $\tilde{\tau}$ using the initial value $\tilde{\mathbf{x}}(t)$, which yields

$$\tilde{\mathbf{x}}(t+\tilde{\tau}) = \mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}\tilde{\mathbf{x}}(t) + \int_0^{\tilde{\tau}} \mathrm{e}^{\tilde{\mathbf{A}}\theta}\tilde{\mathbf{B}}u(t-\theta)\mathrm{d}\theta\,. \tag{1.45}$$

Expressing the integral term and substituting it into Equation (1.31) implies

$$\mathbf{x}_\mathrm{p}(t+\tilde{\tau}) = \mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}\left(\mathbf{x}(t) - \tilde{\mathbf{x}}(t)\right) + \tilde{\mathbf{x}}(t+\tilde{\tau})\,. \tag{1.46}$$

Equations (1.29), (1.32) and (1.46) show that the governing equations of the closed control loop are in fact the same for the MSP and for the FSA in the case of state feedback.

In the case of output feedback, it follows from Equations (1.35), (1.45) and (1.24) that

$$\hat{y}_\mathrm{p}(t+\tilde{\tau}) = y(t) - \tilde{y}(t) + \hat{y}(t+\tilde{\tau})\,. \tag{1.47}$$

Thus, the governing equations for the MSP and the FSA are the same also for output feedback, cf. Equations (1.27), (1.36) and (1.47). This was also pointed out in [8].

The equivalence of the MSP and the FSA can be verified in frequency domain as well. Substitution of Equation (1.37) into Equation (1.39) leads to the following relationship between two concepts:

$$\hat{Z}(s) = \tilde{\mathbf{C}}\mathrm{e}^{-\tilde{\mathbf{A}}\tilde{\tau}}\mathbf{Z}_x(s)\,. \tag{1.48}$$

Then, Equations (1.21) and (1.40) verify the equivalence of the MSP and the FSA.

1.3.3 IMPLEMENTATION ISSUES

Although their equations are equivalent, the MSP and the FSA approaches imply different realizations for the control law. The MSP uses expressions of the estimated output and state in the control laws (1.27) and (1.28), respectively. Meanwhile, FSA replaces these terms with an integral of past control inputs, see Equations (1.31) and (1.35). The role of this integral is crucial in the implementation of the controller.

Realization of the predictor via the right-hand side of Equation (1.37) involves unstable pole-zero cancellation if $\tilde{\mathbf{A}}$ has unstable eigenvalues, hence it is not suitable for stabilizing unstable systems [6, 19]. Thus, implementation of the MSP using $\hat{Z}(s)$ in Equation (1.21) is possible for stable plants only, otherwise unstable pole-zero cancellations may occur. For unstable plants, one may use an integral of control inputs as in FSA, see Equations (1.37) and (1.48).

For FSA, the integral term is typically realized via approximation by numerical quadrature. According to [19, 20, 21, 22], this approximation may lead to a high-frequency instability phenomenon, which introduces additional conditions for safe implementation. These restrictions can be removed by adding a low-pass filter into the controller [8], or by using a digital controller with a sample-and-hold unit [8, 23].

During the implementation of the SP, a similar high-frequency instability phenomenon occurs if the transfer function of the corresponding delay-free closed control loop is proper but not strictly proper. In such cases, additional conditions for safe implementation (also called as practical stability conditions) must be fulfilled [8].

Furthermore, the computation of the matrix exponential $e^{-\tilde{\mathbf{A}}\tilde{\tau}}$ also leads to numerical issues if $\tilde{\mathbf{A}}$ has eigenvalues with large negative real parts. In this case, the so-called unified Smith predictor can be applied to overcome this problem [13, 24], which uses the SP for the stable subsystem and the MSP for the unstable subsystem of the plant.

In connection to controller implementation, the mismatches between the parameters of the internal model and those of the actual system also affect the stability of the closed control loop. Infinitesimal delay mismatches are addressed in [8], while the effect of finite parameter mismatches are analyzed in [17] for the SP and in [18] for the FSA. In addition, there might be additional phenomena (such as noise, nonlinearities and nonsmoothness) that are not modeled by Equation (1.17), but affect the closed-loop dynamics.

1.4 APPLICATION OF OBSERVERS

When the state $\mathbf{x}$ is not (fully) available for feedback, one may either use the output feedback controllers (1.18), (1.27) and (1.36) or a state observer. Here, we consider the case of a Luenberger observer with parameters given by the matrix $\mathbf{L} \in \mathbb{R}^{n\times 1}$. Based on the order of observation and prediction, we can distinguish observer-predictor and predictor-observer representations [13, 14, 15]. Here, we discuss these representations briefly in time domain (more details, frequency-domain description and block diagrams are given in [13]).

In the observer-predictor representation [14], first a state observer is employed that uses the output y:

$$\dot{\mathbf{x}}_\mathrm{O}(t) = \left(\tilde{\mathbf{A}} + \mathbf{L}\tilde{\mathbf{C}}\right)\mathbf{x}_\mathrm{O}(t) + \tilde{\mathbf{B}}u(t-\tilde{\tau}) - \mathbf{L}y(t)\,. \tag{1.49}$$

Then, the observed state $\mathbf{x}_\mathrm{O}$ is introduced into a state predictor:

$$\mathbf{x}_\mathrm{p}(t+\tilde{\tau}) = \mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}\mathbf{x}_\mathrm{O}(t) + \int_0^{\tilde{\tau}} \mathrm{e}^{\tilde{\mathbf{A}}\theta}\tilde{\mathbf{B}}u(t-\theta)\mathrm{d}\theta\,, \tag{1.50}$$

cf. Equation (1.31). Using the predicted state $\mathbf{x}_\mathrm{p}(t+\tilde{\tau})$, control law (1.32) can be used (where $\tilde{\mathbf{A}} + \tilde{\mathbf{B}}\mathbf{K}$ and $\tilde{\mathbf{A}} + \mathbf{L}\tilde{\mathbf{C}}$ are Hurwitz). Recall that the MSP is usually formulated for output feedback using an output predictor, while the FSA is more commonly formulated for state feedback using a state predictor. Therefore, the observer-predictor realization with a state predictor is more closely related to FSA [13, 15]. Note, however, that the integral term in Equation (1.50) could be replaced using Equation (1.45), which leads to

$$\mathbf{x}_\mathrm{p}(t+\tilde{\tau}) = \mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}\left(\mathbf{x}_\mathrm{O}(t) - \tilde{\mathbf{x}}(t)\right) + \tilde{\mathbf{x}}(t+\tilde{\tau})\,. \tag{1.51}$$

Thus, the control law (1.29) of the MSP could also be used with the observed state $\mathbf{x}_\mathrm{O}(t)$ instead of the actual $\mathbf{x}(t)$. This way, the observer-predictor representation also allows one to realize the extension of the MSP to state feedback.

In the predictor-observer representation [13, 15], first the output predictor (1.47) is used, then the predicted output is utilized in a state observer:

$$\dot{\hat{\mathbf{x}}}_\mathrm{J}(t) = \left(\tilde{\mathbf{A}} + \mathbf{L}\tilde{\mathbf{C}}\right)\hat{\mathbf{x}}_\mathrm{J}(t) + \mathrm{e}^{-\tilde{\mathbf{A}}\tilde{\tau}}\tilde{\mathbf{B}}u(t-\tilde{\tau}) - \mathbf{L}\hat{y}_\mathrm{p}(t)\,. \tag{1.52}$$

This way, the future value of the observed state $\hat{\mathbf{x}}_\mathrm{J}$ is directly used by the controller:

$$u(t) = \hat{\mathbf{K}}\hat{\mathbf{x}}_\mathrm{J}(t+\tilde{\tau})\,. \tag{1.53}$$

Since an output predictor is utilized, the predictor-observer realization is said to be more closely related to the control law (1.28) of the MSP [13, 15], where $\mathbf{x}(t) - \tilde{\mathbf{x}}(t) + \hat{\mathbf{x}}(t+\tilde{\tau})$ is replaced by $\hat{\mathbf{x}}_\mathrm{J}(t+\tilde{\tau})$. Note, however, that the output predictor extension (1.35) of FSA could also be used in the predictor-observer representation. In this case, control law (1.32) is applied where $\mathbf{x}_\mathrm{p}(t+\tilde{\tau})$ is replaced by $\mathbf{x}_\mathrm{J}(t+\tilde{\tau}) = \mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}\hat{\mathbf{x}}_\mathrm{J}(t+\tilde{\tau})$.

The relationship between the observer-predictor and the predictor-observer realizations can be established by

$$\hat{\mathbf{x}}_\mathrm{J}(t+\tilde{\tau}) = \mathbf{x}_\mathrm{O}(t) - \tilde{\mathbf{x}}(t) + \mathrm{e}^{-\tilde{\mathbf{A}}\tilde{\tau}}\tilde{\mathbf{x}}(t+\tilde{\tau})\,. \tag{1.54}$$

This can be verified by the differentiation of Equation (1.54) with respect to time and the substitution of Equations (1.49) and (1.52). The order of prediction and observation is therefore interchangeable in the level of equations, but these are two different realizations of the controller. The observer-predictor realization requires a state predictor, while the predictor-observer requires an output predictor.

Table 1.1 Governing equations in time domain for the SP, the MSP and the FSA with state feedback.

Smith Predictor	plant	$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t-\tau)$
	internal model	$\dot{\tilde{\mathbf{x}}}(t) = \tilde{\mathbf{A}}\tilde{\mathbf{x}}(t) + \tilde{\mathbf{B}}u(t-\tilde{\tau})$
	control law	$u(t) = \mathbf{K}(\mathbf{x}(t) - \tilde{\mathbf{x}}(t) + \tilde{\mathbf{x}}(t+\tilde{\tau}))$
	dependence on initial conditions	$u(t) = \mathbf{K}\Big(\mathrm{e}^{\mathbf{A}t}\mathbf{x}_0 + \int_0^t \mathrm{e}^{\mathbf{A}(t-\theta)}\mathbf{B}u(\theta-\tau)\mathrm{d}\theta - \mathrm{e}^{\tilde{\mathbf{A}}t}\tilde{\mathbf{x}}_0 - \int_0^t \mathrm{e}^{\tilde{\mathbf{A}}(t-\theta)}\tilde{\mathbf{B}}u(\theta-\tilde{\tau})\mathrm{d}\theta + \mathrm{e}^{\tilde{\mathbf{A}}(t+\tilde{\tau})}\tilde{\mathbf{x}}_0 + \int_0^{t+\tilde{\tau}} \mathrm{e}^{\tilde{\mathbf{A}}(t+\tilde{\tau}-\theta)}\tilde{\mathbf{B}}u(\theta-\tilde{\tau})\mathrm{d}\theta\Big)$
	perfect internal model $\tilde{\mathbf{A}} = \mathbf{A}$, $\tilde{\mathbf{B}} = \mathbf{B}$, $\tilde{\tau} = \tau$	$u(t) = \mathbf{K}\mathbf{x}(t+\tau) + \mathbf{K}\left(\mathrm{e}^{\mathbf{A}t} - \mathrm{e}^{\mathbf{A}(t+\tau)}\right)(\mathbf{x}_0 - \tilde{\mathbf{x}}_0)$
Modified Smith Predictor	plant	$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t-\tau)$
	internal model	$\dot{\tilde{\mathbf{x}}}(t) = \tilde{\mathbf{A}}\tilde{\mathbf{x}}(t) + \tilde{\mathbf{B}}u(t-\tilde{\tau})$
	modified model	$\dot{\hat{\mathbf{x}}}(t) = \tilde{\mathbf{A}}\hat{\mathbf{x}}(t) + \mathrm{e}^{-\tilde{\mathbf{A}}\tilde{\tau}}\tilde{\mathbf{B}}u(t-\tilde{\tau})$
	control law	$u(t) = \hat{\mathbf{K}}(\mathbf{x}(t) - \tilde{\mathbf{x}}(t) + \hat{\mathbf{x}}(t+\tilde{\tau})) = \mathbf{K}\left(\mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}(\mathbf{x}(t) - \tilde{\mathbf{x}}(t)) + \tilde{\mathbf{x}}(t+\tilde{\tau})\right)$
	dependence on initial conditions	$u(t) = \mathbf{K}\Big(\mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}\mathrm{e}^{\mathbf{A}t}\mathbf{x}_0 + \mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}\int_0^t \mathrm{e}^{\mathbf{A}(t-\theta)}\mathbf{B}u(\theta-\tau)\mathrm{d}\theta - \mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}\mathrm{e}^{\tilde{\mathbf{A}}t}\tilde{\mathbf{x}}_0 - \mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}\int_0^t \mathrm{e}^{\tilde{\mathbf{A}}(t-\theta)}\tilde{\mathbf{B}}u(\theta-\tilde{\tau})\mathrm{d}\theta + \mathrm{e}^{\tilde{\mathbf{A}}(t+\tilde{\tau})}\tilde{\mathbf{x}}_0 + \int_0^{t+\tilde{\tau}} \mathrm{e}^{\tilde{\mathbf{A}}(t+\tilde{\tau}-\theta)}\tilde{\mathbf{B}}u(\theta-\tilde{\tau})\mathrm{d}\theta\Big)$
	perfect internal model $\tilde{\mathbf{A}} = \mathbf{A}$, $\tilde{\mathbf{B}} = \mathbf{B}$, $\tilde{\tau} = \tau$	$u(t) = \mathbf{K}\mathbf{x}(t+\tau)$
Finite Spectrum Assignment	plant	$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t-\tau)$
	internal model	$\dot{\mathbf{x}}_{\mathrm{p}}(t) = \tilde{\mathbf{A}}\mathbf{x}_{\mathrm{p}}(t) + \tilde{\mathbf{B}}u(t-\tilde{\tau})$
	state prediction	$\mathbf{x}_{\mathrm{p}}(t+\tilde{\tau}) = \mathrm{e}^{\tilde{\mathbf{A}}\tilde{\tau}}\mathbf{x}(t) + \int_0^{\tilde{\tau}} \mathrm{e}^{\tilde{\mathbf{A}}\theta}\tilde{\mathbf{B}}u(t-\theta)\mathrm{d}\theta$
	control law	$u(t) = \mathbf{K}\mathbf{x}_{\mathrm{p}}(t+\tilde{\tau})$
	perfect internal model $\tilde{\mathbf{A}} = \mathbf{A}$, $\tilde{\mathbf{B}} = \mathbf{B}$, $\tilde{\tau} = \tau$	$u(t) = \mathbf{K}\mathbf{x}(t+\tau)$

Table 1.2 Governing equations in frequency domain for the SP, the MSP and the FSA with output feedback.

<table>
<tr><td rowspan="3">Smith Predictor</td><td>plant</td><td>$P(s) = \left[\begin{array}{c|c} \mathbf{A} & \mathbf{B} \\ \hline \mathbf{C} & 0 \end{array}\right] = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$
$Y(s) = P(s)\mathrm{e}^{-s\tau}U(s)$</td></tr>
<tr><td>internal model</td><td>$\tilde{P}(s) = \left[\begin{array}{c|c} \tilde{\mathbf{A}} & \tilde{\mathbf{B}} \\ \hline \tilde{\mathbf{C}} & 0 \end{array}\right] = \tilde{\mathbf{C}}(s\mathbf{I} - \tilde{\mathbf{A}})^{-1}\tilde{\mathbf{B}}$
$\tilde{Y}(s) = \tilde{P}(s)\mathrm{e}^{-s\tilde{\tau}}U(s)$</td></tr>
<tr><td>control law</td><td>$Z(s) = \tilde{P}(s) - \tilde{P}(s)\mathrm{e}^{-s\tilde{\tau}}$
$U(s) = C(s)\,(R(s) - (Y(s) + Z(s)U(s)))$</td></tr>
<tr><td rowspan="4">Modified Smith Predictor</td><td>plant</td><td>$P(s) = \left[\begin{array}{c|c} \mathbf{A} & \mathbf{B} \\ \hline \mathbf{C} & 0 \end{array}\right] = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$
$Y(s) = P(s)\mathrm{e}^{-s\tau}U(s)$</td></tr>
<tr><td>internal model</td><td>$\tilde{P}(s) = \left[\begin{array}{c|c} \tilde{\mathbf{A}} & \tilde{\mathbf{B}} \\ \hline \tilde{\mathbf{C}} & 0 \end{array}\right] = \tilde{\mathbf{C}}(s\mathbf{I} - \tilde{\mathbf{A}})^{-1}\tilde{\mathbf{B}}$
$\tilde{Y}(s) = \tilde{P}(s)\mathrm{e}^{-s\tilde{\tau}}U(s)$</td></tr>
<tr><td>modified model</td><td>$\hat{P}(s) = \left[\begin{array}{c|c} \tilde{\mathbf{A}} & \tilde{\mathbf{B}} \\ \hline \tilde{\mathbf{C}}\mathrm{e}^{-\tilde{\mathbf{A}}\tilde{\tau}} & 0 \end{array}\right] = \tilde{\mathbf{C}}\mathrm{e}^{-\tilde{\mathbf{A}}\tilde{\tau}}(s\mathbf{I} - \tilde{\mathbf{A}})^{-1}\tilde{\mathbf{B}}$
$\hat{Y}(s) = \hat{P}(s)\mathrm{e}^{-s\tilde{\tau}}U(s)$</td></tr>
<tr><td>control law</td><td>$\hat{Z}(s) = \hat{P}(s) - \tilde{P}(s)\mathrm{e}^{-s\tilde{\tau}}$
$U(s) = C(s)\Big(R(s) - \Big(Y(s) + \hat{Z}(s)U(s)\Big)\Big)$</td></tr>
<tr><td rowspan="3">Finite Spectrum Assignment</td><td>plant</td><td>$P(s) = \left[\begin{array}{c|c} \mathbf{A} & \mathbf{B} \\ \hline \mathbf{C} & 0 \end{array}\right] = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$
$Y(s) = P(s)\mathrm{e}^{-s\tau}U(s)$</td></tr>
<tr><td>internal model</td><td>$\tilde{P}(s) = \left[\begin{array}{c|c} \tilde{\mathbf{A}} & \tilde{\mathbf{B}} \\ \hline \tilde{\mathbf{C}} & 0 \end{array}\right] = \tilde{\mathbf{C}}(s\mathbf{I} - \tilde{\mathbf{A}})^{-1}\tilde{\mathbf{B}}$</td></tr>
<tr><td>control law</td><td>$\mathbf{Z}_x(s) = \int_0^{\tilde{\tau}} \mathrm{e}^{-(s\mathbf{I}-\tilde{\mathbf{A}})\theta}\tilde{\mathbf{B}}\mathrm{d}\theta$
$\hat{Y}_{\mathrm{p}}(s)\mathrm{e}^{s\tilde{\tau}} = Y(s) + \tilde{\mathbf{C}}\mathrm{e}^{-\tilde{\mathbf{A}}\tilde{\tau}}\mathbf{Z}_x(s)U(s)$
$U(s) = C(s)\Big(R(s) - \hat{Y}_{\mathrm{p}}(s)\mathrm{e}^{s\tilde{\tau}}\Big)$</td></tr>
</table>

1.5 SUMMARY AND CONCLUSIONS

Predictor feedback controllers were discussed by describing the Smith predictor, the modified Smith predictor, and the finite spectrum assignment technique both in frequency and time domain. A detailed comparison was made between these techniques by considering (extensions to) both state and output feedback. The governing equations for state feedback in time domain are summarized in Table 1.1, while those for output feedback in frequency domain are collected in Table 1.2. It was shown that the governing equations of the MSP and the FSA are practically equivalent, while the approach to formulate and realize these controllers is different.

The performance of the SP, the MSP and the FSA depends on the accuracy of the internal model used for prediction. In the case of a perfectly matching internal model, the MSP and the FSA techniques are able to stabilize unstable systems, while this is not possible for the SP due to its disturbance response and the effect of initial conditions.

Sensitivity to infinitesimal implementation inaccuracies in the control law can be observed for the MSP and the FSA. In order to avoid unstable pole-zero cancellations, these predictors should be implemented using integrals of past control inputs instead of realizing models given by differential equations. The approximation of these integrals by numerical quadratures affects stability. Thus, either conditions for safe implementation must be met or low-pass filters or digital controllers must be used. Furthermore, an additional term must be taken into account for the MSP to ensure zero static error.

Finally, when the state of the plant is not available, observers must be utilized. Based on the order of prediction and observation, observer-predictor and predictor-observer realizations are possible that use state and output predictors, respectively. After realizing the observers and overcoming the difficulties of implementation, predictor feedback controllers become efficient tools for compensating the destabilizing effect of feedback delays.

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